MATRIX REVIEW SHEET

DEFINITION:
Matrix **A** is a rectangular array of numbers (elements), arranged in m rows and n columns, referred to as an m x n matrix.
Each element is indicated by its position (i^{th} row, j^{th} column) as $\mathbf{a}_{\mathbf{i,\,j}}$.

Ex: $\begin{bmatrix} \mathbf{3} & \mathbf{-6} & \mathbf{11} \\ \mathbf{0} & \mathbf{4} & \mathbf{-2} \end{bmatrix}$ is a ___ x ___ matrix. The element -6 is $a_{\_\_,\_\_}$ and $a_{2,2}$ = _______.

TRANSPOSE: The transpose of a matrix $\mathbf{(A^T)}$ is found by interchanging rows and columns.

Ex. If **A =** $\begin{bmatrix} \mathbf{3} & \mathbf{-6} & \mathbf{11} \\ \mathbf{0} & \mathbf{4} & \mathbf{-2} \end{bmatrix}$, then $\mathbf{A^T}$ **=**

COMPLEX CONJUGATE: The complex conjugate of a matrix ($\bar{\mathbf{A}}$) is found by replacing each element of the matrix with its complex conjugate.

TRANSPOSE OF THE CONJUGATE: The transpose of the conjugate **(A*)** is also called the **adjoint** of the matrix.

Ex: B = $\begin{bmatrix} \mathbf{3+2i} & \mathbf{-2} \\ \mathbf{0} & \mathbf{-2-i} \end{bmatrix}$

What is $\bar{\mathbf{B}}$?

What is **B*** ?

MATRIX PROPERTIES & OPERATIONS

Equality of Matrices: A = B if each element of A is equal to the corresponding element in B

Addition of Matrices: A + B A and B must be the same size. Add corresponding elements of the matrices.

Scalar Multiplication: A scalar is any real number (or variable representing a real number). Multiply each element of the matrix by the scalar.

Matrix Multiplication: Matrices must be size compatible.

Ex: If A is 3 X 2 and B is 2 x 5 then AB is possible because (3 x 2) (2 x 4) inner values are equal and resulting matrix is measure of outer values (3 x 4). =

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 2 & 1 & 3 \\ -1 & -2 & 4 & 0 \end{bmatrix}$ Find AB =

Inverse of a Matrix: You cannot divide by a matrix. The alternative is to multiply by the inverse of a matrix ($\mathbf{A}^{-1}$). Only square (n x n) matrices could have an inverse, but not all square matrices have inverses. Use your calculator to find the inverse of a matrix.

Ex. Find the inverse (if possible) of: $A = \begin{bmatrix} 1 & 3 \\ 5 & 8 \end{bmatrix}$. $A^{-1} =$

Ex. Find the inverse (if possible) of: $B = \begin{bmatrix} 2 & 4 \\ 10 & 20 \end{bmatrix}$. $B^{-1} =$

Matrix Functions: A matrix function has each element as a function of t or another variable:

Ex: $A = \begin{bmatrix} \sin t & t^2 - 1 \\ \sqrt{t} & \ln(t) \end{bmatrix}$

For a matrix function A:

A is **continuous** if each element of A is **continuous**.
A is **differentiable** if each element of A is **differentiable**.
A is **integrable** if each element of A is **integrable**.