ANALYSIS OF VARIANCE (ANOVA) – section 9.1 & 9.2
(Single Factor Design)

ANOVA used to test the equality of more than two means. It is an extension of the hypothesis test for two means we studied in Chapter 8. The test value is computed by means of an ANOVA table.

SOURCE	SUM OF SQUARES (SS)	Degrees of Freedom (df)	MEAN SQUARE (MS)	F VALUE or F RATIO
Between treatments or means	**SSTr** or SSB	**k – 1**	**MSTr** or MSB	**F (test statistic)**
Within Error or means	**SSE** or SSW	**n – k**	**MSE** or MSW	not used here
Total	**SST** or SSTOT	**n - 1**	not used here	not used here

FIND:

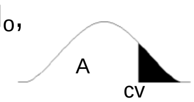
1. **k** = number of groups with n_i values in group i
2. **n** = total number of data values
3. $\bar{\bar{X}}$ **(grand mean)** = $\dfrac{\sum x}{n}$ **(mean of all the data values)**
4. Find mean $\bar{x}_i$ and standard deviation s_i for each group separately
5. **SSTr** = $\sum\left[n_i\left(\bar{x}_i - \bar{\bar{x}}\right)^2\right]$
6. **SSE** = $\sum\left[(n_i - 1)s_i^2\right]$
7. **SSTOT** = **SSTr + SSE** (used as check value only)
8. **Degrees of Freedom** = see table above
9. **MSTr** = $\dfrac{SSTr}{k-1}$ **MSW** = $\dfrac{SSE}{n-k}$
10. **F (test statistic)** = $\dfrac{MSTr}{MSE}$

HYPOTHESIS TEST FOR ANOVA

H_o: $\mu_1 = \mu_2 = \ldots = \mu_k$ (k is number of populations) (treated as right-tailed test)
H_a: At least two μ's are different

Use given α level.
Critical value: from Table VIII on pages 573-578. **Use:** $\begin{cases} \text{given } \alpha \\ \text{df numerator} = k-1 \\ \text{df denominator} = n-k \end{cases}$
Test value: use F-ratio for ANOVA table above.

Decision Rule: If **test statistic > critical value**, then reject H_o,
otherwise accept H_O



ANOVA Example #1: In planning for future staffing, the ages of 19 hospital staff members were analyzed. Three groups (nurses, doctors, and x-ray techs) were chosen. At $\alpha = .05$, can it be concluded that the average ages of the three groups differ?

Nurses	Doctors	X-ray Techs
23	60	33
25	36	28
26	29	35
35	56	29
42	32	23
22	54	41
	58	

Sum of data $\sum x_i$	173	325	189
Mean $\bar{x}$	28.8	46.4	31.5
St. deviation s_i	7.94	13.46	6.25
No. of values n_i	6	7	6

k =
n =
$\bar{\bar{x}}$ =

SOURCE	SUM OF SQUARES (SS)	Degrees of Freedom (df)	MEAN SQUARE (MS)	F VALUE
Between means				
Within a mean				not used
Total			not used	not used

H_o:

H_a:

α **=**

Critical value: **Test value:**

Decision:

Calculations:

ANOVA Example #2: Workers are randomly assigned to four machines on an assembly line. The number of defective parts produced by each worker for one day is recorded. At α=.01, test the claim that the mean number of defective parts produced by the workers is the same

Machine 1	Machine 2	Machine 3	Machine 4
3	3	5	9
2	3	7	9
0	2	8	8
4	0	6	8
4	1	4	1
3	4	5	2
5	7	5	0

Sum of data $\sum x_i$	21	20	40	
Mean $\bar{x}$	3	2.85	5.71	
std. dev. s_i	1.63	2.26	1.38	
No. of values n_i	7	7	7	

k =
n =
$\bar{x}$ =

SOURCE	SUM OF SQUARES (SS)	Degrees of Freedom (df)	MEAN SQUARE (MS)	F VALUE
Between means				
Within a mean				not used
Total			not used	not used

H_o:

H_a:

α =

Critical value: **Test value:**

Decision:

Calculations: