Vectors and the Geometry of Space
Calculus III – Chapter 9 Formulas

VECTORS given $u = \langle u_1, u_2, u_3 \rangle = u_1 i + u_2 j + u_3 k$ and $v = \langle v_1, v_2, v_3 \rangle = v_1 i + v_2 j + v_3 k$

magnitude of vector v: $|v| = \sqrt{v_1^2 + v_2^2 + v_3^2}$ **unit vector =** $\dfrac{v}{|v|}$

dot product: $u \bullet v = |u||v|\cos\theta = u_1 v_1 + u_2 v_2 + u_3 v_3$ if $u \bullet v = 0$, then u and v are orthogonal

scalar projection of v onto u: $\text{comp}_u v = \dfrac{u \bullet v}{|u|}$ **vector projection of v onto u:** $\text{proj}_u v = \dfrac{u \bullet v}{|u|^2} u$

cross product: $u \times v = |u||v|\sin\theta = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

remember : $v \times u = -(u \times v)$

$u \times v\ \ ' = u' \times v + u \times v'$

area of a parallelogram = $u \times v$ **area of a triangle** = $\dfrac{1}{2}\ u \times v$

scalar triple product (box product or area of a parallelepiped): $\left| u \bullet\ \ v \times w\ \right| = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

LINES, PLANES, DISTANCES

distance between points: $|P_1P_2| = \sqrt{\ x_2 - x_1\ ^2 +\ y_2 - y_1\ ^2 +\ z_2 - z_1\ ^2}$

midpoint: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2} \right)$

vector equation for a line through P_0 and parallel to v: $r(t) = r_0 + tv$ (r_0 is position vector of P_0)

parametric equations for a line through P_0 and parallel to $v = \langle a, b, c \rangle$

$$x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$$

symmetric equations for a line through P_0 and parallel to $v = \langle a, b, c \rangle$

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

line segment from r_0 to r_1 (connecting tips of vectors r_0 and r_1): $r(t) = (1 - t)r_0 + tr_1 \quad 0 \le t \le 1$

vector equation of a plane determined by point P_0 (vector r_0 ends at P_0) and normal vector n:

$$n \bullet (r - r_0) = 0 \qquad \text{or} \qquad n \bullet r = n \bullet r_0$$

scalar equation of a plane where $n = \langle a, b, c \rangle,\ r = \langle x, y, z \rangle,\ r_0 = \langle x_0, y_0, z_0 \rangle$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

linear equation of a plane: $ax + by + cz + d = 0$

distance from point $P_1(x_1, y_1, z_1)$ to plane ax + by + cz + d = 0: $D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

distance from line (with points Q and R) to point P (let $a = \overrightarrow{QR}$ and $b = \overrightarrow{QP}$) $d = \dfrac{|a \times b|}{|a|}$

COORDINATES, MISCELLANEOUS

cylindrical to rectangular coordinates: $x = r\cos\theta \quad y = r\sin\theta \quad z = z$

rectangular to cylindrical coordinates: $r^2 = x^2 + y^2 \quad \tan\theta = \dfrac{y}{x} \quad z = z$

spherical to rectangular coordinates: $x = \rho\sin\phi\cos\theta \quad y = \rho\sin\phi\sin\theta \quad z = \rho\cos\phi$

rectangular to spherical coordinates: $\rho^2 = x^2 + y^2 + z^2$

equation of a sphere with center (h, k, l) and radius r: $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$