SOLVING SYSTEMS OF LINEAR EQUATIONS (sec 6.3)

1. **Algebraically** (elimination method) (p. 271)
   Set up and do as in section 6.2, working with any two equations at a time. The goal is to get two equations with two variables. When you get a value for one variable, you can substitute that into **any** of the equations used to get the other variables.

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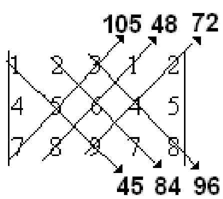
2. **Finding the determinant of a 3 x 3 matrix** (not in book – alternative method on page 273) This is called the **Basket Weave Method**. Remember a determinant can be any real number.

   a) Set up the matrix then recopy the first two columns at the end of the matrix, making 5 columns.

   Example: $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ recopy first two columns to get: $\begin{vmatrix} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{vmatrix}$

   b) Like a 2 x 2 matrix, multiply the upper left to lower right diagonal numbers and add totals
   c) Like a 2 x 2 matrix, multiply the lower left to upper right diagonal numbers and subtract totals

Example:



Determinant = 45 + 84 + 96 – 105 – 48 – 72 = 0

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3. **Determinants (Cramer's Method)** page 273 – similar to that in section 6.2

   a) set up equations in standard form:
   b) find the determinant of the coefficient matrix
   c) for the three variables, replace the column for that variable with the constants and find that determinant. Then divide by the determinant of the coefficient matrix.

$$x = \frac{\begin{vmatrix} \text{const} & y & z \\ \text{const} & y & z \\ \text{const} & y & z \end{vmatrix}}{\det \text{of coefs}} \qquad y = \frac{\begin{vmatrix} x & \text{const} & z \\ x & \text{const} & z \\ x & \text{const} & z \end{vmatrix}}{\det \text{of coefs}} \qquad z = \frac{\begin{vmatrix} x & y & \text{const} \\ x & y & \text{const} \\ x & y & \text{const} \end{vmatrix}}{\det \text{of coefs}}$$

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4. Matrices – rref method (not in book) – an extension of method discussed in sec 6.2

   a) Arrange equations in standard form.
      If the system has 3 equations and 3 variables, enter data into a 3 x 4 matrix.
      If the system has 4 equations and 4 variables, enter data into a 4 x 5 matrix, etc.
   b) Use 2^{ND} – QUIT to save the matrix.
   c) Use: 2^{nd} – MATRIX – MATH – A:rref and press enter
   d) Use: 2^{nd} - MATRIX – NAMES – 1:A and press enter, then type a) and press enter.
      Answer appears in last column in the order you entered the variables.

   NOTE: The process is the same no matter how many equations/variables you have.

5. Matrices – Method #2 (page 277)

a) Set up the coefficients only as a 3 x 3 or 4 x 4 matrix called A. The size is based on the number of equations/variables you have.
b) Set up the constants only as a single column matrix size 3 x 1 or 4 x 1 called B.
c) Use 2nd – MATRIX – NAMES to do the formula:

$[A]^{-1}*[B]$ and solution will appear as a column with values of variables in the order you entered them into A

NOTE: the formula is the same no matter how many equations/variables you have.

POLYNOMIAL DATA MODELING – sec 6.4

1. **Fitting data exactly to given points:**

* For a <u>linear model</u> and points (x_1, y_1) and (x_2, y_2) set up and rref matrix: $\left[\begin{array}{cc|c} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{array}\right]$

Last column of solution is (top to bottom): b, a in $P(x) = ax+b$

* For a <u>quadratic model</u> and points (x_1, y_1) through (x_3, y_3) set up and rref matrix: $\left[\begin{array}{ccc|c} 1 & x_1 & x_1^2 & y_1 \\ 1 & x_2 & x_2^2 & y_2 \\ 1 & x_3 & x_3^2 & y_3 \end{array}\right]$

Last column of solution is (top to bottom): c, b, a in $P(x) = ax^2 + bx + c$

* For a <u>cubic model</u> and points (x_1, y_1) through (x_4, y_4) set up and rref matrix: $\left[\begin{array}{cccc|c} 1 & x_1 & x_1^2 & x_1^3 & y_1 \\ 1 & x_2 & x_2^2 & x_1^3 & y_2 \\ 1 & x_3 & x_3^2 & x_3^3 & y_3 \\ 1 & x_4 & x_4^2 & x_4^3 & y_4 \end{array}\right]$

Last column of solution is (top to bottom): d, c, b, a in $P(x) = ax^3 + bx^2 + cx + d$

2. **For a best fitting model** (least squares polynomial), enter the x values in L1 and y values in L2.

For a quadratic equation use: QuadReg L1, L2

For a cubic equation use: CubicReg L1, L2

For a quartic equation use: QuartReg L1, L2

* NOTE: There is no short cut for finding the equation of a circle given three points. Use the method in text on page 293