# Partial Derivatives
## Calculus III - Chapter 11 Formulas

**Equation of tangent plane to surface of $z = f(x,y)$ at point $P(x_0, y_0, z_0)$ is:** $z = z_0 = f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0)$

**Linear or tangent plane approximation of f at (a,b) is:** $f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

**total differential:** $dz = f_x(x,y)dx + f_y(x,y)dy = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

**Chain Rule {for $z = f(x,y)$ with $x = g(t)$ and $y = h(t)$}:** $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

**Chain Rule {for $z = f(x,y)$ with $x = g(s,t)$ and $y = h(s,t)$}:** $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$ and $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$

**Implicit Differentiation:** $\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{F_x}{F_y}$ other forms on page 785

**Directional Derivative in direction of unit vector $u = \langle a,b \rangle$:** $D_u f(x,y) = f_x(x,y)a + f_y(x,y)b = \nabla f(x,y) \cdot u$

**Directional Derivative when u makes angle $\theta$ with $+x$ axis and $u = \langle \cos\theta, \sin\theta \rangle$:** $D_u f(x,y) = f_x(x,y)\cos\theta + f_y(x,y)\sin\theta$

**Gradient of f:** $\nabla f(x,y,z) = \langle f_x, f_y, f_z \rangle = \frac{\partial f}{\partial x}i + \frac{\partial f}{\partial y}j + \frac{\partial f}{\partial z}k$

**Equation of tangent plane to level surface of $F(x,y,z) = k$ at point $P(x_0, y_0, z_0)$ is:**

$$F_x(x_0,y_0,z_0)(x - x_0) + F_y(x_0,y_0,z_0)(y - y_0) + F_z(x_0,y_0,z_0)(z - z_0) = 0$$