Infinite Series

$\lim_{n\to\infty} \frac{\ln(n)}{n} = 0$	$\lim_{n\to\infty} \sqrt[n]{n} = 1$	$\lim_{n\to\infty} x^{1/n} = 1$ for $x > 0$	$\lim_{n\to\infty} x^n = 0$ for $\lvert x\rvert < 1$	$\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$	$\lim_{n\to\infty} \frac{x^n}{n!} = 0$

geometric (x^n)**: in form** $\sum ar^{n-1}$ **converges to** $\frac{a}{1-r}$ **if** $\lvert r\rvert < 1$**; diverges if** $\lvert r\rvert \geq 1$

harmonic $\left(\frac{1}{n}\right)$**: diverges; alternating harmonic: converges**

telescoping $\left(\frac{1}{n(n+1)}\right)$**: converges**

p-series $\left(\frac{1}{n^p}\right)$**: conv. if** $p > 1$**; div. if** $p < 1$**; alt. p – series: abs. conv. if** $p > 1$**, cond. conv. if** $0 \leq p \leq 1$

Nth Term Test for Divergence: take $\lim_{n\to\infty} a_n \begin{cases} \text{if } \neq 0 \text{ or does not exist, } a_n \text{ diverges} \\ \text{if } = 0, \text{ try something else} \end{cases}$

Integral Test: take $\int_1^\infty a_n\, dn \begin{cases} \text{if } = \pm\infty, \ a_n \text{ diverges} \\ \text{if } \neq \pm\infty, \ a_n \text{ converges} \end{cases}$

Direct or Basic Comparison Test: use a_n to find b_n $\begin{cases} \text{if } a_n < b_n \text{ and } b_n \text{ converges, then } a_n \text{ converges} \\ \text{if } a_n > b_n \text{ and } b_n \text{ diverges, then } a_n \text{ diverges} \\ \text{any other situation, try something else} \end{cases}$

Limit Comparison Test: use a_n to find b_n, take $\lim_{n\to\infty} \frac{a_n}{b_n} \begin{cases} \text{if } > 0 \text{ and } \neq \infty, \ a_n \text{ and } b_n \text{ act the same} \\ \text{if } = 0 \text{ and } b_n \text{ convergent, } a_n \text{ convergent} \\ \text{if } = \infty \text{ and } b_n \text{ divergent, } a_n \text{ divergent} \\ \text{if } < 0, \text{ try something else} \end{cases}$

Ratio Test: take $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert \begin{cases} \text{if } < 1, \ a_n \text{ abs. converges} \\ \text{if } > 1, \ a_n \text{ diverges} \\ \text{if } = 1, \text{ try something else} \end{cases}$

Root test: take $\lim_{n\to\infty} \sqrt[n]{a_n} \begin{cases} \text{if } < 1, \ a_n \text{ converges} \\ \text{if } > 1, \ a_n \text{ diverges} \\ \text{if } = 1, \text{ try something else} \end{cases}$

For Alternating Series:

Try ACT by using any positive term test:
- If $\lvert a_n\rvert$ converges: series has absolute conv.
- If $\lvert a_n\rvert$ not convergent then try Leibnitz:
- If all parts true: series has conditional conv.
- If not all true: series is divergent

Absolute convergence theorem for alternating series (ACT): if $\lvert a_n\rvert$ **converges, then** a_n **converges**

Rearrangement thm: if a_n converges absolutely and b_n is rearrangement of a_n, then b_n converges.

Alternating Series Test (Leibnitz thm): $\sum(-1)^n b_n$ **converges if all are true** $\begin{cases} \text{all } b_n \text{ are positive} \\ b_{n+1} \leq b_n \\ \lim_{n\to\infty} b_n \to 0 \end{cases}$

What test do I use?
(In general: look for patterns, reduce with algebra, write out terms)

1. **Is there a pattern?**
   a. Geometric
   b. Harmonic
   c. Telescoping
   d. p-series

2. **Is it a positive term series?**
   a. Has n! or n as an exponent: try **Ratio test**
   b. Entire series is to a power of n: try **Root test**
   c. a_n is easy to integrate: try **Integral test**
   d. There are 'extra' constants or series is close to a pattern: try **Direct/Basic Comparison Test**
   e. At a loss or no results from Direct/Basic Comparison Test: try **Limit Comparison Test**

3. **Is it an alternating term series?**
   a. First try the Absolute Convergence Theorem (ACT)
   b. If not convergent under ACT, try Alternating Series (Leibnitz)Test

4. **Still at a loss?**
   a. Rearrange terms and try Rearrangement Theorem
   b. Use Nth Term Test for Divergence

Estimating the Sum of a Series: $s_n + \int_{n+1}^{\infty} f(x)\,dx \le s \le s_n + \int_{n}^{\infty} f(x)\,dx$

POWER SERIES (centered at a)

form is $\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \ldots$

Test for convergence by using Root or RatioTest (usually) and Alternating Series Test

Only three possible outcomes:

1. Result of Test is ∞, then series **converges only when x = a**

2. Result of Test is **zero**, then series **converges (absolutely) for all values of x**

3. Result of Test is **some expression involving |x|,** then series has **interval of convergence** (check each endpoint of interval by subbing into original power series).