FINDING THE n^{th} ROOTS OF A COMPLEX NUMBER
(worksheet)

Example: $y^{(4)} + 16 = 0 \Rightarrow r^4 = -16 \Rightarrow r = \sqrt[4]{-16}$

In general:

$$\sqrt[n]{r} = (r)^{1/n} = \sqrt[n]{|r|}\left(\cos\left(\frac{\pi}{n} + \frac{2\pi}{n}k\right) + i\sin\left(\frac{\pi}{n} + \frac{2\pi}{n}k\right)\right) \text{ for } k = 0, 1, 2, \ldots (n-1)$$

Here:

$$\sqrt[4]{-16} = (-16)^{1/4} = \sqrt[4]{16}\left(\cos\left(\frac{\pi}{4} + \frac{2\pi}{4}k\right) + i\sin\left(\frac{\pi}{4} + \frac{2\pi}{4}k\right)\right) \text{ for } k = 0, 1, 2, 3$$

k = 0	$2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \sqrt{2} + \sqrt{2}\,i$
k = 1	$2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = 2\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = -\sqrt{2} + \sqrt{2}\,i$
k = 2	
k = 3	

General Solution:

$y =$